Signaling and Optimal Sorting

by

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Abstract

I consider educational signaling of inherent ability that facilitates sorting of individuals between sectors. More able individuals are more productive in the *primary sector*, and less able individuals are more productive in the *secondary sector*. I find signaling *may increase* but never *maximizes* welfare, and is more likely to increase welfare the greater is productivity in the secondary sector, and, possibly, the lower is productivity in the primary sector. Consistent with recent increased undergraduate enrollment in the U.S, excessive signaling occurs by *less able individuals*. If education increases human capital, total welfare likely increases although more individuals may over-invest in education.

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1. Introduction

Scholars and pundits continue to debate whether education increases productivity (Leonhardt, 2011). In a study for the Social Science Research Council, Arum *et al*. (2011) find that 36% of U.S. college students learn very little after four years. One economist (Bryan Caplan, 2011) recently argued that little is learned in college, higher education is essentially a signal, and welfare would increase if students spent fewer years in school.[1]

One problem for the signaling hypothesis is that potentially cheaper screening mechanisms should exist. Gary Becker suggested "Surely a year on the job or a systematic interview and applicant-testing program must be a much cheaper and effective way to screen."[2] However, in the European Union, applicant test responses are personal data, and some countries (Greece and France in particular) make employment tests difficult (Perritt, 2008). In the U.S., limits to the use of applicant testing are the U.S. Supreme Court decision *Griggs v. Duke Power* (1971) and subsequent court rulings and legislation. O'Keefe and Vedder (2008) argue that employers now focus on very narrow subjects directly related to jobs in applicant testing. After Griggs there has been less use of general aptitude and intelligence tests that might help employers determine an individual's ability to learn and advance at a firm. O'Keefe and Vedder suggest that educational signaling has substituted for employment testing since nothing prevents individuals from obtaining educational credentials to try to demonstrate ability to firms.[3]

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[1] In a new book, *The Case Against Education* (2018), Caplan argues that 80% of the effect of education is signaling.
[2] Becker (1993, p.8).
[3] At least one National Football League team, the Eagles under coach Chip Kelly, asked potential draftees many questions about their college academics (why a player chose his major, what was his hardest class, etc.). The team believed that college graduation is more than proof of intelligence, and is a signal that an individual is committed to achieving goals (Clark, 2014).

Assuming that signaling occurs, I will address two issues. First, who (if anyone) over-invests in education when it is a signal? The typical signaling[4] model (Spence, 1974a, and Riley, 2001, for example) implies that more able individuals over-invest in education in order to sort themselves from those who are less able. Although such an argument might explain more individuals receiving, for example, an MBA in order to sort themselves, the recent significant increase in undergraduate enrollment in the U.S. cannot have come from the more able unless relatively few of them previously enrolled in college. From 1995 to 2010, the 18-24 year old population in the U.S. increased from 24.28 million to 27.16 million, a 12% increase.[5] Yet the number enrolled in college in that period rose from 14.26 million to 21.02 million, a 47% increase.[6] As will be demonstrated, this large increase in undergraduate enrollment is consistent with the idea that relatively low ability individuals may over-invest in education in order to distinguish themselves from those with even lower ability.

Second, and the main focus herein, is whether signaling may improve welfare by allowing individuals to be allocated to jobs where they are most productive. Kenneth Arrow was asked about education being wasteful in the signaling model. Arrow responded "It permits the

[4] Some claim evidence of fairly rapid employer learning of skills implies a low value of educational signaling (Altonji and Pierret, 1997, and Lange, 2007). However, Habermalz (2011) shows such a conclusion is unwarranted. Habermalz demonstrates that signaling is not necessary with very rapid employer learning. With very slow employer learning, signaling would not likely be worthwhile due to the short period for which ability would be revealed. With an intermediate length of employer learning, the value of the signal increases with the speed of learning. Estimated returns to education are consistent with theoretical results that imply that signaling occurs. Waldman (2016) combines educational signaling with signaling to other firms by promotion decisions. He argues that studies such as that by Lange (2007) underestimate the returns to signaling because of the assumption such returns are low late in an employee's work life. Waldman finds that educational signaling has potential returns later in individuals' careers when there is asymmetric learning (other firms know less about productivity of a worker than does the worker's employer) and promotion signaling. For simplicity, I ignore different jobs at the same employer.

[5] Source: US Bureau of the Census Current Population Reports.

[6] Source: *statista*: http://www.statista.com/statistics/183995/us-college-enrollment-and-projections-in-public-and-private-institutions/

sorting. That in itself may be socially gainful. And Spence's model in its form doesn't fully convey all its implications. It implies that an individual will be productive no matter what."[7]

In this paper, I add to the formal analysis of the sorting effect of educational signaling.[8] I develop a theoretical model that avoids the argument about how much education increases human capital/productivity by generally assuming there is no direct effect of education on productivity. I make such an assumption in order to consider if education can increase welfare by sorting individuals to jobs. Later (Section Five) I allow for the possibility that education increases human capital. Thus, the main contribution of this paper is to add to the literature on the welfare effects of educational signaling when the social gain from education is that it allows for an improved sorting of individuals between jobs.

I assume there may be a social gain from education as a signal in that it allows individuals to be sorted to jobs where their productivity is the highest. Signaling models generally ignore the sorting gain from education and assume a continuous signal. Such models have either a few types of individuals in terms of ability (typically two) or a continuum of individual ability. In those models, an individual of one ability type may be perfectly identified via signaling. In contrast, I assume a continuum of ability, with individuals getting either zero education (beyond some level assumed to already be attained by all) or one additional level of education. Thus, different ability types attain the same education level---zero or the additional level.

[7] *The Region* (1995, p.8).
[8] Willis (1986) mentions the possibility signaling may be socially worthwhile if the allocation of individuals to jobs matters. For early work that considered sorting, see Spence (1974a) and Stiglitz (1975), both of which are discussed in Section Two below.

The truth lies somewhere between the usual assumption that one can be perfectly identified and my assumption of many ability types getting the same level of education. The question is which assumption fits the evidence. The number enrolling in college increasing much faster than the school age population is consistent with the hypothesis that those in the lower ability range of college matriculants try to distinguish themselves from those who are even less able, which occurs with a discrete level of education.[9] One result I find, which does not appear to have been previously considered, is that signaling may increase welfare---relative to a world with no signaling---but be socially excessive relative to the number who should signal to maximize welfare.

The outline of the rest of this paper is as follows. In Section Two, I compare my approach to other signaling models. My model is developed in Section Three. In Section Four, I consider numerical examples to see when welfare increases with signaling. The possibility that education increases human capital is considered in Section Five, and Section Six contains a summary.

2. Comparisons to previous research

The signaling literature was developed by Spence (1974a) and extended in important ways by Riley (1975, 1979). Those authors assumed a continuous signal. This led Spence (1974a) to argue that multiple signaling equilibria could exist (in addition to possible pooling

[9] With a continuum of ability and a continuous signal, all but the least able over-invest in education when it is only a signal and there is no sorting gain from education (Riley, 2001). Such a model requires all to invest more in education if, say, the difference in education costs between ability levels is reduced. It is hard to reconcile that result with the large recent increase in enrollment in the U.S., which suggests that only some individuals have increased the level of their educational signal.

equilibria). Riley (1975) suggested that only one signaling equilibrium (the Pareto dominant one) might survive experimentation by buyers, and even that equilibrium might not survive further experimentation. Later, Riley (1979) suggested that a reactive equilibrium[10] (again the Pareto dominant one) would survive. As will be shown in Section Three, there is one signaling equilibrium when there is only one level of the signal and a continuum of individuals.

The closest approach to my model is in Lazear (1986). He considers a two-sector model in which productivity is proportional to individual ability in the primary sector, with an alternative that pays the same for all. Information is symmetric: all are uninformed about productivity in the primary sector. If productivity is measured, one is paid a wage equal to productivity minus measurement cost in the primary sector. Measurement of productivity in Lazear's model is equivalent to an accurate test. If measurement is not possible, and expected productivity exceeds alternative earnings, all work in salary firms in the primary sector. However, those with productivity less than that in the alternative should work in the latter sector. Sorting via measurement cost achieves a higher output but at a cost. Lazear considers the asymmetric information case---primary sector employers do not know individual ability, and individuals know their own ability---but not with an alternative sector. Then there is no possible social gain from testing/signaling since there is no sorting.

Spence (2002) considers a model based on Lazear (1986) when there is asymmetric information and no alternative sector. Productivity is learned at a cost (the same for all). Thus, in contrast to his original model (Spence, 1974a), the signal is not a continuous variable, and it directly reveals ability. The net benefit of the signal/test is positively related to productivity,

[10] In a reactive equilibrium, agents consider possible reactions by other agents before the former take actions.

which is necessary for a signaling equilibrium since cost is the same for all. Separating and pooling equilibria exist simultaneously. As in Lazear (1986), when there is no alternative sector, so all individuals have the same productivity at any firm, there is no social gain from signaling.

In Regev (2012), more able individuals have a higher benefit but the same cost as the less able from signaling. Two discrete signals are possible, and equilibrium is in mixed strategies. In her model, a pass fail test also exists, there is no sorting gain from education, and education enhances productivity. In contrast, I assume no additional test, there is a sorting gain from education, and (generally) education does not affect productivity. My results are driven by a semi-separating equilibrium which differs from that in Regev who has the same type choosing different education levels. I have a continuum of types, where the more able types choose the discrete level of education, and less able types choose no education. Expected productivity could be higher in either sector, and there is a potential social return to signaling from sorting individuals to the sector in which their productivity is highest as suggested by Arrow (*The Region*, 1995).

Additionally, I differ from Regev (2012) in that I consider a discrete signal that is not a test, and that has a cost that is inversely related to ability. The signal implicitly reveals who has productivity above a certain level.[11] In my model, over-investment in education as a signal is not by the more able. Instead, as in Waldman (2016), too many at lower ability levels signal since all with the same education level get the same wage. Clearly the assumption of a binary signal leads to excessive signaling by less able and not more able individuals. Excessive signaling by less

[11] Arcidiacono *et al*. (2010) suggest that college may directly reveal aspects of individual ability to employers. I assume no direct revelation is possible. If direct revelation were possible, education would essentially act as a test as in Lazear (1986), Perri (1994), and Spence (2002).

able individuals is consistent with the significant recent increase in college enrollment in the U.S. discussed in Section One.

Hopkins (2012) considered signaling with more than one job. He analyzed a matching model in which the signal identifies individuals' inherent ability, and may increase the value of the match. In the Hopkins model, individuals do not consider the benefit of their signal to potential partners, and may underinvest in the signal. As in the model herein, the quality of firms is observable in Hopkins' model. Thus, signaling in a matching environment may involve externalities that have opposite effects. In a job allocation model, there are no offsetting effects. However, the social and private returns to signaling are not the same, so signaling may be efficient or inefficient.[12]

Hoppe *et al.* (2009) consider a marriage market with symmetric ignorance of the quality of men and women, both of whom signal their quality. Hoppe *et al*. also find both positive and negative externalities.[13] Symmetric ignorance seems less relevant for the job model herein. Prospective employees know about pay and working conditions at a firm, which reveal a good deal about a firm's quality.

Finally, two early attempts to deal with sorting are Spence (1974a) and Stiglitz (1975). In an appendix, Spence considers two different sectors when there is a potential social gain from sorting individuals to the correct sector. With two types of individuals, Spence finds that welfare

[12] Wolpin (1977) considers the use of signaling (more precisely, screening) by firms to reduce misallocation between and within firms in a model in which a larger variance in skill of labor inputs reduces expected output. He argues too much or too little screening could occur. There is only one occupation in his model, so there is no gain from assigning individuals to different types of jobs. An individual is equally valuable at any firm, given the skill variance at that firm.

[13] For example, a man who signals imposes a negative externality on lower quality men since the latter have one less high quality woman with whom to match. Conversely, a man who signals presents a positive externality for women by his presence.

depends on whether the more able are more or less productive than the less able in the secondary sector. Spence (1974a) apparently did not view the job allocation problem as having great importance because the results were ambiguous.[14] I also find signaling may or may not improve wealth. However, my results are more precise than in Spence in terms of what the effect of signaling on wealth will be.

Stiglitz considers a model with two types of individuals (call them more and less able), two types of jobs (call them skilled and unskilled), and a cost of screening[15] individuals for ability that is the same for all and is perfectly accurate. Stiglitz finds that, if screening cost is too high, there exists a semi-pooling equilibrium in which a fraction of the more able screen. In this case, *too little screening* occurs if the more able are less productive than the less able in the unskilled job. The reason for too little screening is that the less able subsidize the more able in the unskilled job where the wage equals expected productivity.

In my model, there is a continuum of individual ability, and all have the same productivity in the secondary sector. In Section Three, I show that welfare depends on the productivity of all in either sector,[16] and in which sector individuals would be employed absent signaling. I find that too much signaling always occurs. However, signaling may still increase

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[14] In his book, Spence says he considered the problem of job allocation "...largely in response to suggestions and questions raised by Zvi Griliches and George Stigler..." (1974a, p.152). He also devoted only four of thirty-seven pages in a journal article on signaling to job allocation (Spence, 1974b).

[15] Typically, when individuals incur cost to identify themselves it is called signaling. Firms incurring such cost is called screening.

[16] Waldman (2016) has a model that is similar to mine in that there is one education level, there are two wages (for those with and without education, the former hired in the primary sector, and the latter in the secondary sector in my model), signaling cost is inversely related to ability, and too much signaling occurs by those who are *less able*. Waldman is interested in how signaling affects job assignment when said assignment is a signal to alternative employers of individual ability, and when education is directly productive. In particular, in his model, educational signaling reduces the distortion in the promotion decision when promotion is also a signal. I assume no promotions, and that education is generally not directly productive. I am concerned with the welfare effects of signaling which depend on in which sector individuals would be if educational signaling did not occur.

welfare. It is noteworthy that, in his Nobel lecture, Spence (2002) considered the case upon which I focus, that in which there is a discrete signal (or test in Spence's case).

3. A two sector model with signaling

3.1. Setup

I consider a world with two sectors. In the *primary sector*, individual ability, x, affects productivity. One can be think of this as the sector with skilled jobs. In the *secondary sector*, x is less important for productivity than in the primary sector. For simplicity, I assume x has no effect on productivity in the secondary sector. There all have productivity of ω, with $\omega > 0$. Information is asymmetric. An individual knows his value of x. Employers know the distribution of x.

I assume x is continuously distributed on the interval $[0, x_{max}]$ with a density of $f(x)$, a cumulative density of $F(x)$, and with the expected value of x denoted by $E(x)$. I normalize the number of individuals to one. Productivity in the primary sector is vx, with $v > 0$. Also, I assume $vx_{max} > \omega$ so individuals with $x > \omega/v$ are more productive in the primary sector, and those with $x < \omega/v$ are more productive in the secondary sector. Note that a larger v is consistent with skill-biased technological change since a larger v implies that individuals are more valuable in the primary sector where more ability means greater productivity.

At some cost, $\mathcal{C}(x)$, an individual can acquire a signal which can be thought of as a discrete level of education. The signal is *not* an accurate test. As in the standard signaling model

(Spence, 1974a) it is assumed that $C(x)$ is inversely related to x.[17] Those with $x \geq x^*$ acquire education, and will be hired in the primary sector and paid a wage equal to expected productivity of those with $x \geq x^*$ as firms in the primary sector infer who would optimally acquire education. Those with $x < x^*$ do not acquire education. They are hired in the secondary sector and paid ω.

So that those who do not signal are hired in the secondary sector, it must be the case that the expected productivity of those who do not signal---$v\mathrm{E}(x|x<x^*)$---is less than ω. If

$v\mathrm{E}(x|x<x^*) > \omega$, then all would be in the primary sector, with or without signaling. The secondary sector would be inactive. In that case, all signaling would do is redistribute income. Absent signaling, all would be paid $\mathrm{E}(x)$. With signaling, those who signal would be paid $v\mathrm{E}(x|x>x^*)$, and those who do not signal would be paid $v\mathrm{E}(x|x<x^*)$. This is the basic case considered by Spence (1974) when signaling must decrease welfare. Thus, in order to consider the possibility signaling can increase welfare via sorting, I assume that $\omega > v\mathrm{E}(x|x<x^*)$.

Above, I assumed that $vx_{max} > \omega$. A stronger assumption implies that signaling always occurs. With no signaling, all would be in either the primary or secondary sector depending on where expected productivity is higher. With $v\mathrm{E}(x)$ the expected productivity if all were in the primary sector, the wage with no signaling would equal $max(v\mathrm{E}(x),\omega)$. I assume that

[17] One criticism of signaling models is the assumption that signaling cost and ability are inversely related (Weis, 1983, and Regev, 2012). I believe such an assumption is reasonable. More able individuals require less effort to obtain a given level of education. Also, less study time required for the more able to obtain education implies lower foregone earnings for them. In a paper that assumes education simply increases human capital, Becker *et al.* (2010) focus on what they consider nontraditional costs of college such as the difficulty involved. They emphasize the variation in these costs across individuals, and argue that individuals with greater ability have lower costs of schooling.

$vx_{max} > max(v\mathrm{E}(x), \omega) + C(x_{max})$. With an additional assumption regarding out-of-equilibrium beliefs (see the last paragraph in this sub-section), some will always signal.

The timing of the game is as follows. First, each individual's value of x is drawn from $F(x)$. Second, all individuals simultaneously decide whether to obtain education. Third, firms in the two sectors make wage offers. With some individuals obtaining education, primary sector firms will only make wage offers to those with education, and will pay a wage equal to the expected productivity of those who firms believe would optimally choose to acquire education.[18] Those who do not obtain education will be offered a wage of ω in the secondary sector. Fourth, individuals who choose education accept offers from the primary sector. Others accept employment in the secondary sector.[19]

I consider Perfect Bayesian equilibria where it is also assumed that an action off the equilibrium path is by one with the lowest cost of such an action. This implies signaling will always occur. If no signaling occurs, and all are paid the same wage, one with $x = x_{max}$ will deviate from this equilibrium, signal, and earn vx_{max}, which exceeds earnings with no signaling plus signaling cost for these individuals. Thus, there are always some individuals who will signal. As noted above, those who signal will be hired in the primary sector and paid a wage

[18] Other equilibria are possible. For example, if all are in the secondary sector, those with the highest ability have an incentive to deviate from this equilibrium, provided primary sector firms have out-of-equilibrium beliefs that the most able would be the most likely to deviate from the old equilibrium. Those with $x = x_{max}$ who applied to the primary sector without signaling would then be paid vx_{max}, which exceeds ω. However, if primary sector firms offered a wage of vx_{max} to all who applied, then all would apply, and the new equilibrium would not be stable.

[19] Suppose individuals incorrectly estimate the number who would signal, and thereby understate or overstate what the wage would be in the primary sector. Depending on the direction of the estimation error, welfare could be higher or lower than with no error. For analysis of when individuals misjudge what wages or prices will be, see Akerlof and Tong (2013), Akerlof and Shiller (2015), and Perri (2016).

equal to expected productivity of those who would find it optimal to signal, those with $x \geq x^*$. Now x^* is derived from the condition for one to be indifferent to being in either sector:

$$vE(x|x \geq x^*) - C(x^*) = \omega. \tag{1}$$

3.2 Efficiency

Let welfare with signaling be denoted by Ω. Individuals with $x < x^*$ are employed in the secondary sector where each has productivity equal to ω. Total productivity of those in the primary sector is $v \int_{x^*}^{x_{max}} x\, f(x) dx$, and the total cost of signaling for these individuals is $\int_{x^*}^{x_{max}} C(x) f(x) dx$. Thus:

$$\Omega = v \int_{x^*}^{x_{max}} x\, f(x) dx + F(x^*)\omega - \int_{x^*}^{x_{max}} C(x) f(x) dx. \tag{2}$$

Proposition One. Too many individuals signal.

Proof. Differentiate Ω with respect to x^*:

$$\frac{\partial \Omega}{\partial x^*} = f(x^*)[\omega + C(x^*) - vx^*]. \tag{3}$$

If x^* maximizes welfare, $\frac{\partial \Omega}{\partial x^*} = 0$. Using $\frac{\partial \Omega}{\partial x^*} = 0$, the second-order condition becomes $f(x^*)[\frac{\partial C(x^*)}{\partial x^*} - v]$, which is negative since C is inversely related to x. Let $\tilde{x}^*$ denote the value of x^* that maximizes Ω. Using *eq*.(1), $\frac{\partial \Omega}{\partial x^*} = f(x^*)v[E(x|x \geq x^*) - x^*] > 0$. Thus, $x^* < \tilde{x}^*$ so too many signal. □

Productivity in the primary sector of the marginal individual who signals is vx^*. The private return to signaling is $vE(x|x \geq x^*)$ which exceeds vx^* so too many individuals signal.

3.3 Welfare when all would be in the secondary sector absent signaling

If $v\mathrm{E}(x) < \omega$, then, absent signaling, all would be in the secondary sector.[20] In this case, the social gain from signaling is that it leads to employment of some in the primary sector who are more productive there than in the secondary sector.

Proposition Two. When all would be in the secondary sector absent signaling, signaling always increases welfare.

Proof. Signaling increases welfare if $\Omega > \omega$. With $\mathrm{E}(x|x \geq x^*) = \left(\frac{1}{1-F(x*)}\right)\int_{x*}^{x_{max}} x\, f(x)dx$, and

$\mathrm{E}(\mathcal{C}|x \geq x^*) = \left(\frac{1}{1-F(x*)}\right) \int_{x*}^{x_{max}} \mathcal{C}(x)f(x)\mathrm{d}x$, signaling increases welfare if:

$$[1- F(x^*)][\, v\mathrm{E}(x|x \geq x^*) - \mathrm{E}(\mathcal{C}|x \geq x^*) - \omega] > 0. \tag{4}$$

Ineq.(4) indicates that signaling increases welfare if the number who signal, 1-$F(x^*)$, multiplied by the following is positive: for those who signal, the average productivity in the primary sector, minus their average cost of signaling, minus what their output would be in the secondary sector---where all would be employed absent signaling. Using *eq*.(1), *ineq*.(4) becomes:

$$[1- F(x^*)][\mathcal{C}(x^*) - \mathrm{E}(\mathcal{C}|x \geq x^*)] > 0, \tag{4'}$$

which is true since $\mathcal{C}$ is inversely related to x. □

[20] Bickhchandani, Hirshleifer, and Riley (2013) consider a model with a primary job and an outside opportunity that pays more than the expected productivity of individuals in the primary job. They assume two types of individuals and a continuous signal so, as is usually the case, any excessive investment in education is by the more able. I consider a continuum of types and a discrete signal.

Proposition Two should be evident: when all would be in the secondary sector absent signaling, those who choose to signal must be better off since they could have stayed in the secondary sector and earned a wage equal to their productivity, ω. The expected output gain per person from moving individuals from the secondary sector to the primary sector is $v\text{E}(x|x \geq x^*)$ minus ω, and the expected cost is $\text{E}(\mathcal{C}|x \geq x^*)$. The expected output gain equals the cost of signaling for the marginal individual, which exceeds the average cost of signaling. Thus, signaling necessarily increases welfare *if* the alternative to signaling is that all would be employed in the secondary sector.

An increase in ω or a decrease in v decreases the productivity difference between the primary and secondary sectors. Thus, a larger ω or a smaller v implies there is less gain from signaling when, absent signaling, all would be employed in the secondary sector. However, in this case, signaling *always* increases welfare. Since an increase in ω or a decrease in v increases the likelihood all would be in the secondary sector absent signaling, this is one reason why a larger ω or a smaller v may make it more likely that signaling increases welfare.

Excessive education occurs because individuals are not perfectly sorted via the educational signal. Although there obviously are multiple levels of education---different degrees, quality of schools, majors, and performance---it seems reasonable that there are fewer discernible levels of education than there are types of individuals, and that wages do not fully adjust to reflect all of the observable measures of education.

Since signaling always occurs in my model, it is not possible to know where individuals would be employed absent signaling. Recent technical change implies a larger value for v, thus making it more likely that all would be in the primary sector absent signaling via education.

However, the results in this sub-section are still important for the debate on the social value of education when education is primarily a signal. I show that educational signaling *always* increases welfare when the alternative to educational signaling would be for all to be employed in the secondary sector. What happens when all would be in the primary sector absent signaling is addressed in the next sub-section.

3.4 Welfare when all would be in the primary sector absent signaling

If $v\mathrm{E}(x) > \omega$, all would be in the primary sector absent signaling. Now signaling causes some less able individuals to be employed in the secondary sector and not the primary sector.

Proposition Three. When all would be in the primary sector absent signaling, signaling improves welfare only if inequality (5') holds: $[1- F(x^*)][\mathcal{C}(x^*) - E(\mathcal{C}|x \geq x^*)] > vE(x) - \omega$.

Proof. Now signaling increases welfare if $\Omega > v\mathrm{E}(x)$. Using *eq*.(2), this requires:

$$[1- F(x^*)]\,[v\mathrm{E}(x|x \geq x^*) - \mathrm{E}(\mathcal{C}|x \geq x^*)] + F(x^*)\omega > v\mathrm{E}(x). \tag{5}$$

Using the condition for the determination of x^* (*eq*.(1)), *ineq*.(5) reduces to:

$$[1- F(x^*)][\mathcal{C}(x^*) - \mathrm{E}(\mathcal{C}|x \geq x^*)] > v\mathrm{E}(x) - \omega. \tag{5'}$$

Since both sides of *ineq*.(5') are positive, the inequality need not hold. □

The left side of *ineq*.(5') is the difference in signaling cost between the marginal and average individual who signals times the number who signal. The right side of *ineq*.(5') is the difference in average productivity between individuals in the primary and secondary sectors.

Signaling increases welfare only if this cost difference exceeds the productivity difference.[21] Clearly, if vE(x) - ω is large enough, sorting those with $x < \omega/v$ to the secondary sector is not as socially valuable. Also, the left side of *ineq*.(5') is smaller the larger is average signaling cost, E($\mathcal{C}|x \geq x^*$). When average signaling cost is too high, sorting will not increase welfare.

In this case, the social gain from signaling involves the re-allocation of less able individuals to the secondary sector, and the social cost is the cost of signaling by all other individuals. Thus, there is no particular reason why signaling should increase welfare.

3.5. The effects of exogenous variables on welfare

To find the effects of an exogenous change in signaling cost, let $\mathcal{C}(x) = cg(x)$, with c a positive constant. Now we rewrite the equations that determine x^* and illustrate welfare with signaling, Ω.

$$vE(x|x \geq x^*) - cg(x^*) = \omega, \tag{1'}$$

$$\Omega = v\int_{x^*}^{x_{max}} x\,f(x)dx + F(x^*)\omega - \int_{x^*}^{x_{max}} cg(x)f(x)dx. \tag{2'}$$

Clearly E($x|x \geq x^*$) is positively related to x^*, and $g(x^*)$ is inversely related to x^*, so the left side of *eq*.(1') is positively related to x^*. Then, by inspection of *eq*.(1'), if x^* increases, an exogenous increase in signaling cost (dc > 0), or an increase in ω, or a decrease in v is necessary

[21] Interestingly, although the output gain from signaling in this case is from moving less able individuals from the primary sector to the secondary sector, the condition for welfare to be improved via signaling depends on the difference between *average* productivity in the two sectors.

for *eq*.(1') to hold. Thus, $\frac{\partial x*}{\partial c} > 0$, $\frac{\partial x*}{\partial \omega} > 0$, and $\frac{\partial x*}{\partial v} < 0$. Also, we know from the proof for Proposition One that $\frac{\partial \Omega}{\partial x*} > 0$. Thus:

Proposition Four. A larger ω increases welfare with signaling

Proof. Differentiating Ω from *eq*.(2'):

$$\frac{\partial \Omega}{\partial \omega} = \frac{\partial \Omega}{\partial x*}\frac{\partial x*}{\partial \omega} + F(x^*) > 0. \quad \square \tag{6}$$

An increase in ω increases the number of individuals who do not signal and are employed in the secondary sector, increasing output in that sector. Also, with too many individuals signaling, an increase in ω indirectly raises welfare by reducing the number who signal.

Proposition Five. A larger v and a smaller c increase welfare with signaling only if the direct effects of v and c exceed their indirect effects.

Proof. Again differentiating Ω from *eq*.(2'):

$$\frac{\partial \Omega}{\partial v} = \frac{\partial \Omega}{\partial x*}\frac{\partial x*}{\partial v} + \int_{x*}^{x_{max}} x\, f(x)dx \lesseqgtr 0, \tag{7}$$

$$\frac{\partial \Omega}{\partial c} = \frac{\partial \Omega}{\partial x*}\frac{\partial x*}{\partial c} - \int_{x*}^{x_{max}} g(x)f(x)dx \lesseqgtr 0. \quad \square \tag{8}$$

A higher *v* directly increases welfare. However, a higher *v* means that more signal, lowering welfare from signaling as even more individuals signal who should not efficiently do so. Finally, although a larger *c* directly reduces welfare, a larger *c* indirectly raises welfare by reducing the number who signal.

4. Numerical examples

In order to derive explicit solutions for welfare with and without signaling, I assume x is uniformly distributed on the interval [0,1]. Also, I assume $\mathcal{C}(x) = c(1-x)$.[22] With these assumptions, I can solve *eq*.(1) to find the level of x^*:

$$x^* = max\left(0, \frac{2(\omega+c)-v}{v+2c}\right). \quad (9)$$

I focus on the situation when $x^* > 0$. Thus, I assume that $2(\omega + c) > v$ *so* $x^* = \frac{2(\omega+c)-v}{v+2c}$.

Using *eq*.(2'), welfare is now:

$$\Omega = \frac{[1-x*]}{2}[v(1+x*) - c(1-x*)] + \omega x*. \quad (10)$$

Table One shows some examples that illustrate the effect of v, ω, and c on welfare from signaling, Ω. In order for all to be the primary sector absent signaling, it must be the case that $\omega < \frac{v}{2}$. Also, if $x^* = 0$, all would signal and remain in the primary sector---where they would be with no signaling. Signaling cannot increase welfare in that case. Therefore, in Table One, from *eq*.(9), we must have $\frac{v}{2} < \omega + c$.

From Proposition Three, signaling *may* increase welfare when all would be in the primary sector absent signaling, or $\Omega > v\,E(x) = \frac{v}{2}$. In examples 12, 17, and 22 (indicated in bold in Table One), ω is large enough relative to v so that signaling improves welfare. Since in only 3 of

[22] With $x_{max} = 1$, $\mathcal{C}(1) = 0$. This is not important. I could assume some additional fixed cost of signaling for all. As long as $v - \mathcal{C}(1) > \omega$, some will signal.

22 cases does signaling increase welfare, clearly the sorting effect of signaling on welfare depends critically on where individuals would be employed absent signaling.

Additional results involve v and c. Although a larger v does not unambiguously raise welfare, in the 4 cases in Table One when v increases, given ω and c, I find that Ω increases.[23]

Also, although a larger c directly reduces welfare, as noted above, a larger *c may* increase Ω by reducing the number who signal. There are 9 cases in Table One in which c increases, given ω and v. In 6 cases, Ω increases as c increases.[24] In 3 cases, Ω decreases as c increases.[25] Welfare with signaling is more likely to be positively related to signaling cost, c, when v is larger. In that case, relatively more individuals signal inefficiently. Then, by reducing the number who signal, a larger c may increase welfare with signaling.

5. Education increases human capital

So far, I have assumed there is no direct effect of education on individual productivity/human capital. I made that assumption because of the skepticism (for example, Caplan, 2011, 2018) regarding education's effect on productivity. I now consider the possibility that education increases human capital. Again I assume that productivity is unaffected by individual ability---innate or augmented by human capital---in the secondary sector.

[23] These cases are example 7 versus example 3, example 10 versus example 9, example 18 versus example 15, and example 19 versus example 16.
[24] These cases are example 15 versus example 7, example 16 versus example 8, example 17 versus example 9, example 20 versus example 10, example 21 versus example 11, and example 22 versus example 12.
[25] These are example 4 versus example 1, example 5 versus example 2, and example 6 versus example 3.

When all would be in the primary sector absent signaling, I have shown that education as a signal *may* increase welfare. In that case, when education also increases human capital, the effect of education on welfare would still be ambiguous. Thus, I only consider the case when $v(E(x) < \omega$ so all would be in the secondary sector absent signaling. As demonstrated above, this is the case when unproductive education increases welfare.

Proposition Six. If ability is uniformly distributed, a greater productivity effect of education a) increases welfare, but b) causes more to over-invest in education when the alternative to signaling is for all to be in the secondary sector.

Proof. Intuitively, a greater productivity effect of education should increase welfare for a given level of education. However, since more over-invest in education as education is more productive, the welfare effect is uncertain. In the Appendix, I derive the condition for welfare to increase as education is more productive, and prove part a) of Proposition Six. I now prove part b) of Proposition Six.

Suppose education increases human capital/productivity in the primary sector by z: productivity = $v(x + z)$. As before, I assume that x is distributed uniformly on the interval [0,1], and that $\mathcal{C}(x) = c(1\text{-}x)$.

The number who invest in education is 1- x^*. The number who should invest is 1-$\tilde{x}^*$. Thus, the number who over-invest is 1- x^* - (1- $\tilde{x}^*$) = $\tilde{x}^*– x^*$. Now x^* is derived from

$v\left(z + \frac{x+1}{2}\right) – c(1\text{-}x) = \omega$. Assuming that x^* is positive, one finds:

$$x^* = \frac{2(\omega+c)-v(1+2z)}{v+2c} \qquad (11)$$

The welfare-maximizing level of x^* is found when the output gain from an individual moving from the secondary sector to the primary sector equals the individual's cost of signaling $v(z + x) - c(1 - x) = \omega$, or:

$$\tilde{x}^* = \frac{\omega + c - vz}{v + c}. \qquad (12)$$

It is easy to show that $\tilde{x}^* > x^*$.[26] The number who over-invest in education is $\tilde{x}$ - x^*. Using *eqs*.(12 and (13), I then have:

$$\tilde{x}^* - x^* = \frac{v(v - \omega + vz)}{(v + c)(v + 2c)}. \qquad (13)$$

Clearly $\tilde{x}^* - x^*$ is positively related to z so, as more human capital is produced with education, more individuals invest in education who should not do so from a social standpoint, even though total welfare increases (see the Appendix).[27] □

6. Summary

I consider a model in which education is not (usually) productive. In contrast to the usual signaling models (for example, Spence, 1974a, and Riley, 1975, 1979, and 2001), I assume only one level of education is attainable, and there is a continuum of ability. I find signaling always occurs, never maximizes welfare, and may increase welfare. Because educational signaling is optimal for a range of ability, excessive signaling occurs by less able individuals, which is consistent with the evidence of increased enrollment in college in the U.S., but which is not the result in most signaling models.

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[26] I find that $\tilde{x}^* > x^*$ if $z > \frac{\omega - c - v}{v}$. With $v > \omega$, $\tilde{x}^* > x^*$.

[27] Similar results occur if the increase in productivity due to education is proportional to *x*. Note, from *eq*.(12), if $z > \frac{\omega + c}{v}$, $\tilde{x}^* = 0$: all should and would invest in education.

The welfare effects of signaling depend on where individuals would be employed with no signaling. If that is where the more able are more productive, it is questionable whether signaling increases welfare. However, it is possible that, absent signaling, all would be employed in the sector where the less able are more productive. Then I find that signaling always increases welfare.

Additionally, I find that a greater increase in human capital when education occurs [28] does not necessarily increase welfare, but does so at least in the case of a uniform distribution of ability. The reason for the ambiguity in the welfare effect is that there then may be an increase in the number who over-invest in education the more human capital is increased with education.

My main result is that the welfare effect of educational signaling does not simply depend on whether education augments human capital. Of particular importance is what the allocation of individuals to jobs *absent signaling* would be.

[28] Recent research offers differing views of the effect of education on productivity. Eble and Hu (2015) analyze educational reform in China that extended the length of primary school by one year. They find large signaling effects of education. More relevant for my model, which implicitly assumes further education implies a baccalaureate degree, Arteaga (2016) considers the leading Colombian university, which reduced the amount of coursework required for a degree in either economics or business. She finds that human capital accounts for essentially the entire return to education. In contrast, Caplan (2018) claims that 80% of the return to higher education in the U.S. is due to signaling. Finally, Bostwick (2016) considers whether field of study in college serves as a signal of ability. She finds that access to elite schools affects individuals' choice of major at non-elite schools, which is consistent with signaling.

Table One. Welfare from signaling (Ω). With no signaling and all in the primary sector, welfare would equal vE(x) = $\frac{v}{2}$. With no signaling and all in the secondary sector, welfare would equal ω.						
Example #	v	ω	c	x^*	vE(x)= $\frac{v}{2}$	Ω
1	2	.25	1	.125	1	.633
2	2	.5	1	.25	1	.781
3	2	.75	1	.375	1	.945
4	2	.25	1.5	.3	1	.618
5	2	.5	1.5	.4	1	.77
6	2	.75	1.5	.5	1	.938
7	3	.75	1	.1	1.5	1.155
8	3	1	1	.2	1.5	1.32
9	3	1.25	1	.3	1.5	1.495
10	4	1.25	1	.083	2	1.67
11	4	1.5	1	.167	2	1.848

Table One continued.

Example #	v	ω	c	x^*	$v\mathrm{E}(x) = \frac{v}{2}$	Ω
12	4	1.75	1	.25	2	**2.031**
13	3	.25	1.5	.083	1.5	0.88
14	3	.5	1.5	.167	1.5	1.021
15	3	.75	1.5	.25	1.5	1.172
16	3	1	1.5	.333	1.5	1.333
17	3	1.25	1.5	.417	1.5	**1.505**
18	4	.75	1.5	.083	2	1.418
19	4	1	1.5	.143	2	1.551
20	4	1.25	1.5	.214	2	1.713
21	4	1.5	1.5	.286	2	1.883
22	4	1.75	1.5	.357	2	**2.06**

Appendix. Proof of Proposition Six Part A.

This is the case when all would be in the secondary sector absent signaling, so education only as a signal improves welfare. Intuitively, if education also is productive, welfare should be even greater. However, more over-invest in education when education is productive, so it remains to be shown that welfare does increase as education is more productive.

Again, let total welfare with signaling be denoted by Ω. Using *eq*.(2), and adding the productivity effect of education, *z*:

$$\Omega = v\int_{x*}^{x_{max}}(x+z)\,f(x)dx + \omega F(x^*) - \int_{x*}^{x_{max}} \mathcal{C}(x)\mathrm{f}(x)\mathrm{d}x. \quad \text{(A1)}$$

Eq.(A1) can be rewritten as:

$$\Omega = [1- F(x^*)][\, v\mathrm{E}(x+z|x \geq x^*) - \mathrm{E}(\mathcal{C}|x \geq x^*)] + \omega F(x^*). \quad \text{(A2)}$$

However, x^* is determined by:

$$v\mathrm{E}(x+z|x \geq x^*) - \mathcal{C}(x^*) = \omega. \quad \text{(A3)}$$

Using *eq*.(A3), *eq*.(A2) becomes:

$$\Omega = \omega + [1- F(x^*)][\mathcal{C}(x^*) - \mathrm{E}(\mathcal{C}|x \geq x^*)]. \quad \text{(A4)}$$

Using *eq*.(A4), differentiating Ω w.r.t. *z*:

$$\frac{\partial\Omega}{\partial z} = \{[\mathrm{E}(\mathcal{C}|x \geq x^*) - \mathcal{C}(x^*)]\mathrm{f}(x^*) + [1 - \mathrm{F}(x^*)][\frac{\partial(C(x*))}{\partial x*} - \frac{\partial(E(C|x\geq x*))}{\partial x*}]\}\frac{\partial x*}{\partial z}. \quad \text{(A5)}$$

With a greater return to investing in education as *z* increases, clearly $\frac{\partial x*}{\partial z} < 0$. Also, those with more ability, *x*, have lower education cost by assumption, so $\mathcal{C}(x^*) > \mathrm{E}(\mathcal{C}|x \geq x^*)$,

$\frac{\partial(E(C|x\geq x*))}{\partial x*} < 0$, and $\frac{\partial(C(x*))}{\partial x*} < 0$.

Thus the {•} term in *eq*.(A5) is unambiguously negative and $\frac{\partial\Omega}{\partial z} > 0$ if $\left|\frac{\partial(E(C|x\geq x*))}{\partial x*}\right| < \left|\frac{\partial(C(x*))}{\partial x*}\right|$.

With the uniform distribution of *x* in the text, $\frac{\partial(C(x*))}{\partial x*} = -c$, and $\frac{\partial(E(C|x\geq x*))}{\partial x*} = -\frac{c}{2}$, so $\frac{\partial\Omega}{\partial z} > 0$.

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