When to Deploy? The Proximity Fuse in World War Two

Timothy Perri

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Appalachian State U.

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The proximity fuse (PF) was possibly the 2nd most important military invention in WW2.

Widespread deployment of PFs by the U.S. was delayed by approximately 1 year due to the fear the enemy would obtain a dud shell & reverse engineer the PF.

The U.S. faced a cost/benefit problem:

the sooner it deployed PFs the lower its cost until the enemy might deploy them, but the sooner the enemy might deploy PFs.

Did the U.S. wait too long?

Time & contact fuses

Worked well vs. large bomber fleets.

Neither was effective vs. dive bombers or troop concentrations.

Air bursts were 25 times as effective as ground bursts vs. troops, & time fuses worked poorly after a short time.

U.S. developed the 1st PF that could withstand artillery fire---20,000 Gs vs. 3 Gs with the space shuttle---(1st successful test 8/42).

U.S. did not deploy PFs in field artillery until 12/44.

Likely could have deployed them fall '43.

3 places PFs were used before 12/44.

❶ 1/43 ships in Pacific.

In 1943, PFs accounted for 25% of shells fired in the Pacific by the U.S., & 51% of downed Japanese planes.

Over time effectiveness of anti-aircraft artillery (AAA) on ships increased 6 to 7 times with PFs.

Admiral King (CNO) saw this & opposed widespread PF deployment.

❷ After D-day (6/44) on Normandy beach. Not used---no German planes.

❸ In England (8/44) to counter V-1s.

Non-PFs: 500–600 AAA shells to destroy 1 V-1.

PFs: 40–50 shells per V-1 kill.

*** General Hap Arnold (Chief of AAF) insisted PFs be deployed only on the coast.***

HEAVY BOMBERS

Heavy losses over Germany against time & contact fuses.

U.S. flew 719,972 missions.

14,200 aircraft shot down (≈ 2%).

U.S. lost 44% of bombers to fighters & 44% to AAA.

British lost 63% to bombers & 37% to AAA.

Huge #s required to attack major targets.

8/43, 230 U.S. bombers attacked Schweinfurt ball bearing plants.

60 planes lost, 532 men killed or captured.

The plant operated in 2 weeks.

Hap Arnold knew effectiveness of PFs vs. V-1s.

PFs v. ground forces

4/43 Scientists who worked on PFs were asked if they could be used in field artillery.

Production soon began, so deployment could have occurred in a few months.

Vannevar Bush, head of Office of Scientific R&D, convened a panel in 1944 to determine how long it would take to reverse-engineer & deploy a PF from a dud.

27 to 30 months.

Bush met with Admiral King 10/44, & that month the Joint Chiefs approved deployment in field artillery as of 12/25/44.

Actual deployment occurred 12/18/45---Battle of the Bulge.

Army sent officers from the battle to the U.S. to learn optimal fuse settings.

The effect of PFs was devastating. German troops caught in the open were slaughtered.

Airburst from a 155 mm shell could devastate an area 75 yds. in diameter.

Shells cut through logs on top of German bunkers.

German POWs thought either 1) shells were set off by an igniter triggered by earth's magnetism, or 2) U.S. forces had some superior training.

The Model

ℓ = length of time until the war ends

(from when U.S. could widely deploy PFs).

$\ell \in [L_0, L_1]$, with $\ell \sim$ uniformly.

R = the certain length of time after the U.S. deploys PFs on a wide scale that the enemy deploys PFs.

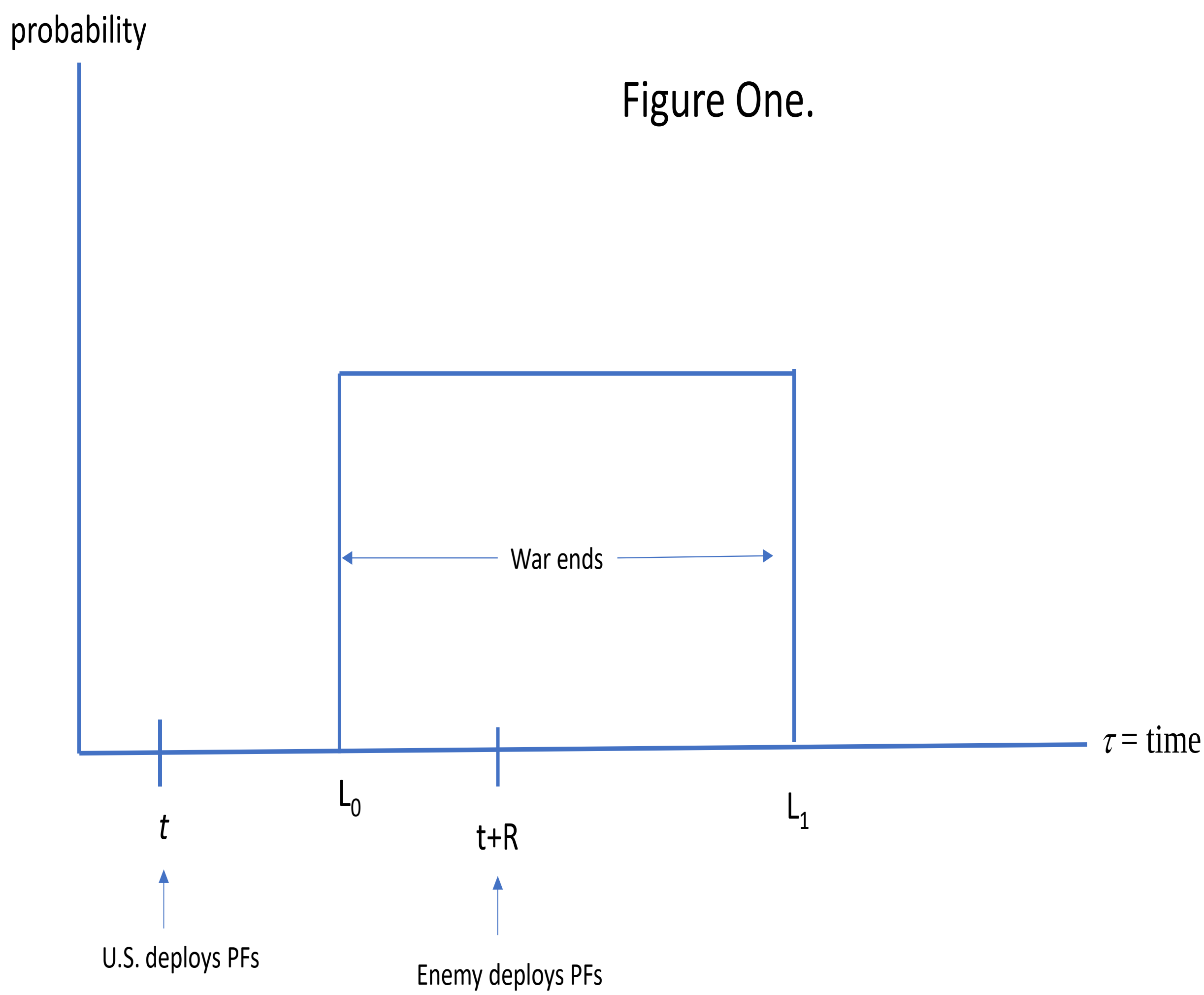
t = time from when the U.S. is 1st capable of deploying PFs on a wide scale until PFs are widely deployed.

2 simplifying assumptions (that fit the evidence):

1) $t < L_0$ & 2) $L_0 < R$.

$\therefore$ Even if $t = 0$ (deploy immediately), the enemy could not deploy until a date when it is possible the war would end.

See Figure One.



probability
Figure One.
War ends
τ = time
t
L_0
t+R
L_1
U.S. deploys PFs
Enemy deploys PFs

Earliest U.S. could widely deploy PFs was likely fall 1943. I use 10/1/43.

PFs for field artillery 1st considered 4/43.

Immediately produced PFs for field artillery.

Approval for their use 10/44 & 1st deployment was 12/44.

Allowing several months for production after 4/43, 6 months seems reasonable for deployment.

A guess is (from 10/1/43):

L_0 = 1.75 (7/1/45) & L_1 = 3.75 (7/1/47).

1) Defeating Germany might have occurred as early as late '44.

2) Marshall (summer of '43) thought invasion of Europe might not happen until '45.

3) Marshall (5/44) was unsure if a) Normandy invasion would succeed, b) the A bomb would be developed, &, c) the A bomb would cause Japanese surrender.

4) FDR in 2/45: European war over in '45; war in Japan could go into '47.

5) Hans Bethe did not work on Manhattan Project---bomb not feasible he felt.

6) Groves: A bomb by 8/45, but it might not work.

7) Days before 1st A bomb was detonated, many at Los Alamos thought it would not work.

$\therefore$ The war ended much sooner than expected (8/45)---the atomic bomb worked & helped induce Japanese surrender.

c_0 = cost of the war (per unit of time) to the U.S. when it had not widely deployed PFs. (mainly used vs. enemy planes attacking U.S. ships).

c_1 = cost to the U.S. (per unit of time) when it widely deployed PFs, & the enemy had not deployed PFs.

c_2 = the cost to the U.S. (per unit of time) when it & the enemy both widely deployed PFs.

Assume $c_1 < c_0 < c_2$.

Clearly $c_1 < c_0$.

Using PFs in field artillery, the U.S. could destroy more enemy personnel & eqt., reducing U.S. casualties.

$c_0 < c_2$ if the gain to the U.S. from deploying PFs in field artillery is < the cost from the enemy using PFs against 1) U.S. ground forces, 2) naval aircraft attacking enemy ships, and 3) bombers attacking enemy territory.

If $c_0 \geq c_2$, $t = 0$: deploy immediately.

U.S. deploys: certainty from t to L_0, + the expected length of time the war ends from L_0 to L_1:

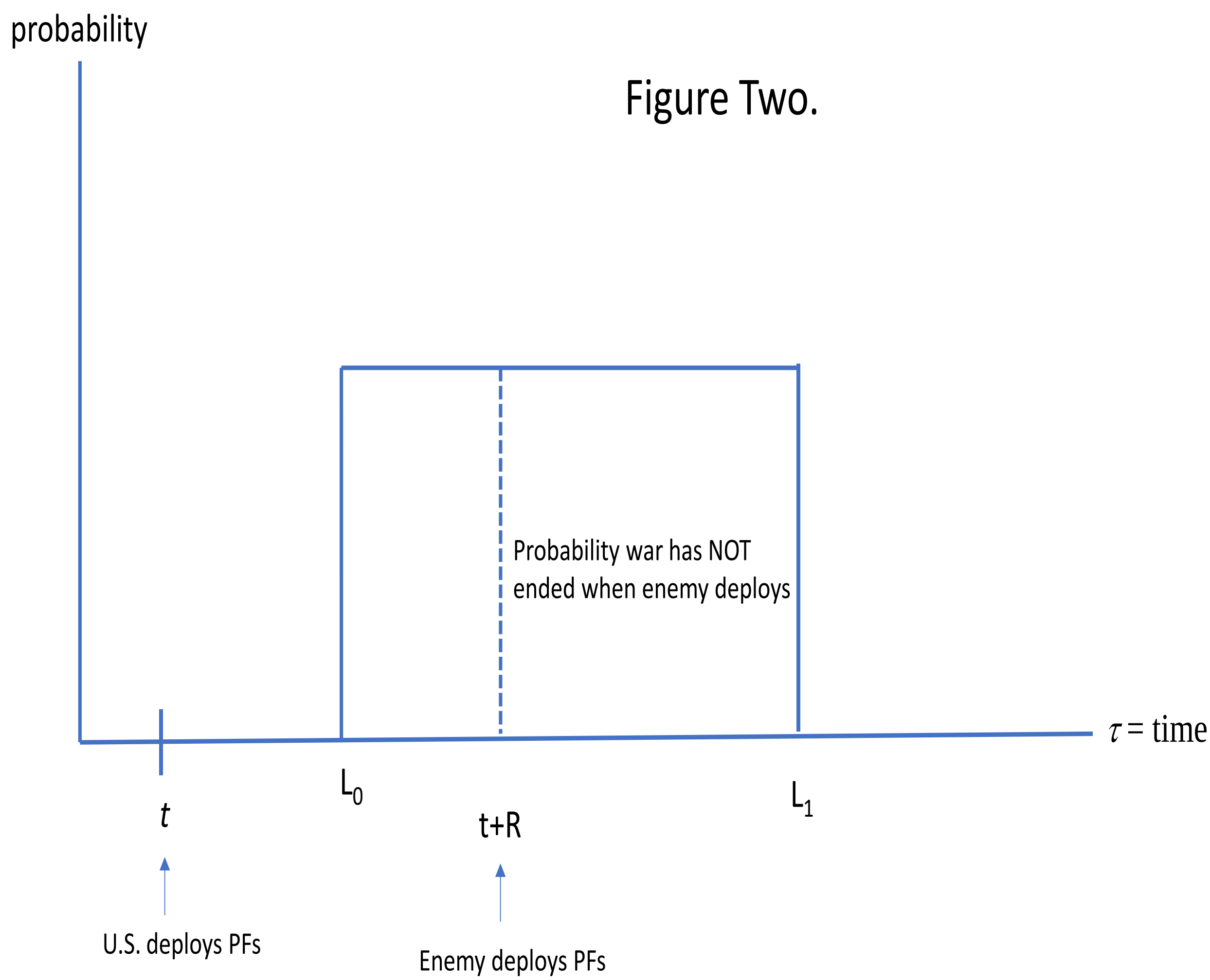
$$L_0 - t + \frac{L_1 - L_0}{2} = \frac{L_0 + L_1 - 2t}{2}. \qquad (1)$$

Enemy's (conditional) expected length of deployment is:

$$\frac{L_1 - t - R}{2}. \qquad (2)$$

Probability war still occurs when enemy deploys:

$$\frac{(L_1 - t - R)}{L_1 - L_0}. \qquad (3)$$


probability
Figure Two.
Probability war has NOT
ended when enemy deploys
τ = time
t
L0
t+R
L1
U.S. deploys PFs
Enemy deploys PFs

Total expected cost to the U.S. = C.

$C = c_0\, t +$

Limited U.S. deployment

$\frac{c_1}{2}[L_0 + L_1 - 2t - \frac{(L_1 - t - R)^2}{L_1 - L_0}] +$

Wide U.S. deployment

$$\frac{c_2}{2}\,\frac{(L_1 - t - R)^2}{(L_1 - L_0)}\;. \qquad (4)$$

U.S. & enemy

deployment

Minimizing C w.r.t. t:

$$\frac{\partial C}{\partial t} = c_0 - c_1 - \frac{(c_2 - c_1)(L_1 - t - R)}{L_1 - L_0} = 0. \qquad (5)$$

$$\frac{\partial^2 C}{\partial t^2} = \frac{(c_2 - c_1)}{L_1 - L_0} > 0. \qquad (6)$$

Gain to the U.S. is lower cost, $c_0 - c_1$, when enemy has not deployed.

Loss to the U.S. is higher expected cost, $\frac{(c_2 - c_1)(L_1 - t - R)}{L_1 - L_0}$, when the enemy has deployed.

I ignore the fact that earlier wide deployment of PFs may have shortened the war.

It would not have affected L_0, but probably reduced L_1.

Thus, I understate the gain to widespread deployment of PFs, & overstate the optimal t, t*.

Deak Parsons, a naval officer who worked on the development of PFs, estimated that each month the U.S. delayed deploying PFs in the Pacific cost it 1 battleship, 3 cruisers, & 1,350 sailors.

The impact of late deployment in field artillery is unknown.

Solving the FOC for t:

$$t^* = L_1 - R - \frac{(L_1 - L_0)(c_0 - c_1)}{c_2 - c_1}. \qquad (7)$$

Clearly $\frac{\partial t*}{\partial c_0} < 0$, $\frac{\partial t*}{\partial c_2} > 0$,

$\frac{\partial t*}{\partial L_0} > 0$, & $\frac{\partial t*}{\partial R} < 0$.

Also $\frac{\partial t*}{\partial c_1} = \{+\}(c_2 - c_0) > 0$,

& $\frac{\partial t*}{\partial L_1} = \{+\}[1 - \frac{(c_0 - c_1)}{c_2 - c_1}] > 0$.

Rewrite equation for t^*

$$t^* = L_1 - R - \frac{(L_1 - L_0)\left(1 - \frac{c_1}{c_0}\right)}{\frac{c_2}{c_0} - \frac{c_1}{c_0}} \qquad (7')$$

$\frac{c_1}{c_0}$ reflects the decreased cost to the U.S. from widespread deployment of PFs.

$\frac{c_2}{c_0}$ reflects the increased cost to the U.S. from the enemy's deployment of PFs, relative to limited U.S. deployment of CFs.

The U.S. use of PFs against ground forces added to the use of PFs on ships (against enemy dive bombers).

Japanese could have used PFs against U.S. ground forces, naval dive bombers, and heavy bombers attacking Japan.

How costly would the use of PFs against U.S. forces have been relative to the lower cost to the U.S. from using PFs against enemy ground forces?

Suppose the enemy's use of PFs against ground forces raised the cost to the U.S. by the same amount the U.S. deployment of PFs against ground forces lowered cost to the U.S.

Then $c_2 \approx c_0$ (if no other use by enemy of PFs).

The use of PFs by the enemy against U.S. heavy bombers and naval dive bombers raises c_2 so that $c_2 > c_0$.

Table One. Determining *t**.

$\frac{c_1}{c_0}$	$\frac{c_2}{c_0}$	t^*
.75	1.1	.071
.75	1.2	.389
.75	1.5	.833
.8	1.1	.167
.8	1.2	.5
.8	1.5	.929
.9	1.1	.5
.9	1.2	.833
.9	1.5	1.167

Table One assumes $L_0 = 1.75$, $L_1 = 3.75$, & $R = 2.25$

If the effect vs. aircraft dominated the effect vs. ground forces, then *t** could have been ≈ 1.

However, the timing of deployment---authorized immediately when *R* estimate was revealed to the military---is suspicious.